(GRAVITATION)

What is gravitation ?

Every body the universe attracts every other body with a force called force of gravitation.

OR

Gravitation is the force of attraction between any two bodies in the universe.

e.g. The attraction between the sun and earth or other planets, force between earth and moon, force of attraction between two particles etc.

Gravitation is the weakest force in the four fundamental force. (strong nuclear, weak nuclear, electromagnetic gravitation). It plays an important role in initiating the birth of stars and in controlling the structure and evolution of the entire universe.

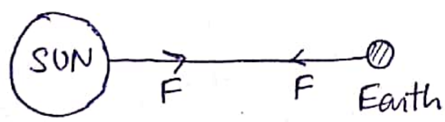
What is Gravity ?

Gravity is a special case of gravitation. If one of the attracting bodies is the earth then the gravitation is called gravity.

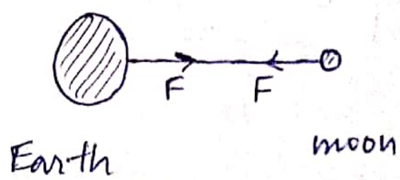
OR

Gravity is a force of attraction between the earth and any object lying on or near its surface.

e.g. A body thrown up falls back on the surface of the earth due to earth's force of gravity.
Atmosphere of earth is also due to the earth's force of gravity.



(Gravitation)



(Gravity)

Scanned by CamScanner

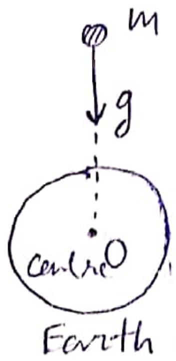
What is Free fall ?

"The motion of a body under the influence of gravity only and no other forces like drag, friction etc acts on the body is called a free fall." A body falls freely only in vacuum. The motion of a small heavy body in air may be taken as a free fall because air resistance on it is very small.

Acceleration due to gravity :-

When a body falls freely towards the surface of the earth. Its velocity continuously increases. Therefore the acceleration produced in a freely falling body under the gravitation pull of the earth is called acceleration due to gravity. It is denoted by g. It is a vector quantity having the direction towards the centre of the earth.

It does not depends on the mass, size and shape of the body.



$$F = G\frac{m \cdot M_e}{R_e^2}$$

$$F = mg \quad \text{(For free fall)}$$

$$\cancel{m}g = G\frac{\cancel{m} \cdot M_e}{R_e^2}$$

$$\boxed{g = \frac{GM_e}{R_e^2}}$$

M_e = mass of earth

R_e = radius of earth

G = Gravitational constant

G, M_e and R_e all are constant. Therefore the value of g is constant at a given place. However it varies from place to place on the surface of the earth. It depends on altitude (height depth, rotation of the earth and shape of the earth.

The value of g near the surface of the earth is $g = 9.8 \ m/s^2$ or $32 \ ft/s^2$

$$\boxed{1 \text{ meter} = 3.28 \text{ ft}}$$

2.56 cm = 1 inch

12 inch = 1 ft

Weight of a body :-

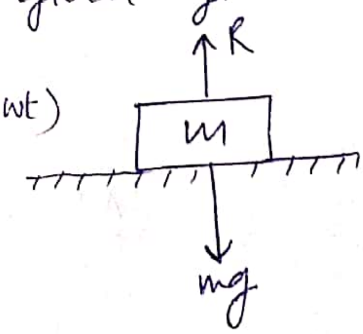
"Weight of a body is defined as the gravitational force with which a body is attracted towards the centre of the earth."

If the g is the acceleration due to gravity at a place then a body of mass of m is attracted towards the centre of the earth with a force equal to mg. at that place.

Hence the weight of a body is given by

$$W = mg$$

(Unit - N, kg-wt) kg.f)



In vector form

$$\boxed{\vec{W} = m\vec{g}}$$

Weight is a vector quantity. It is measured in the unit of force such as Newton.

As the value of g varies from place to place the weight of a body also varies from place to place

$$g_M = \frac{g_e}{6}$$

$$\frac{\vec{W}_m}{\vec{W}_e} = \frac{\cancel{m} \cdot g/6}{\cancel{m} g}$$

$$\boxed{\vec{W}_m = \frac{\vec{W}_e}{6}}$$

g_M = acceleration due to gravity of moon

$\vec{W}_m$ = weight on moon

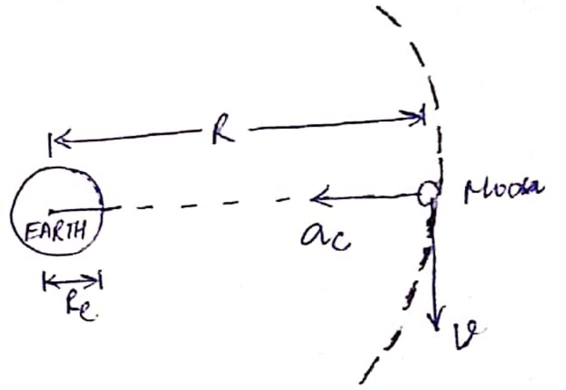
$\vec{W}_e$ = weight on earth

Scanned by CamScanner

Discovery of Newton's law of Gravitation :-

In 1665, Newton assumed that the force which attracts the apple towards the earth might be the same as the force attracting the moon towards the earth. Newton was able to deduce the law of Gravitation By comparing the acceleration due to gravity on the earth with the acceleration required to keep the moon in orbit around the earth, Newton assumed that "the moon revolved around the earth in a circular orbit" of radius $R = 384000$ km

The time period of moon around the earth



$T = 27.3$ days

$T = 27.3 \times 24 \times 60 \times 60$

$T = 27.3 \times 86400$ sec

Velocity of the moon

$$v = \frac{\text{circumference of Orbit}}{\text{Orbit period.}}$$

$$v = \frac{2\pi R}{T} \qquad [\, R = 3.84 \times 10^8 \text{ m} \,]$$

$$v = \frac{2 \times 3.14 \times 3.84 \times 10^8}{27.3 \times 86400}$$

$$v = 1.02 \times 10^3 \text{ m/s}$$

Centripetal acceleration of the moon.

$$a_c = \frac{v^2}{R}$$

$$a_c = \frac{(1.02 \times 10^3)^2}{3.84 \times 10^8} \qquad \boxed{a_c = 2.72 \times 10^{-3} \text{ m/s}^2}$$

Scanned by CamScanner

Acceleration due to gravity at the earth's surface
$g = 9.8\ m/s^2$

clearly $a_c << g$. As Newton' assumed that both acceleration are provided by earth's gravitational attraction.

Hence the acceleration must decrease with distance from the centre of the earth. Therefore Newton proposed that gravitation force should be inversely proportional to the square of distance
If R_e is the radius of the earth. then

$$\frac{a_c}{g} = \frac{\frac{1}{R^2}}{\frac{1}{R_e^2}}$$

$$\frac{a_c}{g} = \left(\frac{R_e}{R}\right)^2$$

Newton knew that $\frac{R_e}{R} = \frac{1}{60}$

$$\frac{a_c}{g} = \left(\frac{1}{60}\right)^2$$

$$a_c = \left(\frac{1}{60}\right)^2 \times 9.8$$

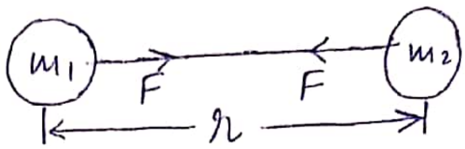
$$\boxed{a_c = 2.72 \times 10^{-3}\ m/s^2}$$

This value is approximately equal to the value obtained earlier. Thus this verifying the inverse square law.
It is called "Newton's moon test"
Newton further analysed that the force of gravitation exerted by an object should be proportional to its mass.
By the third law of motion, The second object should exert equal and opposite force on first one. The force should be proportional to the mass of the second object.

Newton's law of Gravitation :

STATEMENT : "Every particle in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles."

Consider two bodies of masses m_1 and m_2 and distance between them is r.



According to the law of gravitation

$$F \propto m_1 \cdot m_2 \quad \text{———} \quad (1)$$

$$F \propto \frac{1}{r^2} \quad \text{———} \quad (2)$$

From eq^n (1) and (2) we get

$$F \propto \frac{m_1 \cdot m_2}{r^2}$$

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

where G is a constant called universal gravitational constant.

Gravitational constant : If $m_1 = m_2 = 1\,kg$, $r = 1\,m$

$$\boxed{F = G}$$

"The universal gravitational constant may be defined as the force of attraction between two bodies of unit mass each and placed / seperated by a unit distance."

The value of $G = 6.67 \times 10^{-11}\ N\,m^2/kg^2$ in S.I

In C.G.S system $G = 6.67 \times 10^{-8}$ dyne. cm^2/gm^2.

" The value of the gravitational constant G was determined experimentally by "Henry Cavendish" in 1798.

"The value of G does not depend on the nature and size of the bodies. It also does not depend on the nature of the medium between the two bodies. That is why G is called Universal gravitational constant."

Dimension of G ÷ As $F = G\dfrac{m_1 \cdot m_2}{r^2}$

$$G = \frac{F \cdot r^2}{m_1 \cdot m_2}$$

$$G = \frac{[MLT^{-2}] \cdot [L^2]}{[M][M]}$$

$$\boxed{G = [M^{-1} L^3 T^{-2}]}$$

Unit of G ÷ As. $F = G\dfrac{m_1 m_2}{r^2}$

$$G = \frac{F \cdot r^2}{m_1 \cdot m_2}$$

$$G = \frac{N \cdot m^2}{kg \cdot kg}$$

$$\boxed{G = N \cdot m^2/kg^2}$$

In C.G.S $G = \text{dyne}\ cm^2/gm^2$.

Experimental evidences in support of the law of gravitation –

1. The rotation of earth around the sun or motion of moon around the earth is explained on the basis of this law.

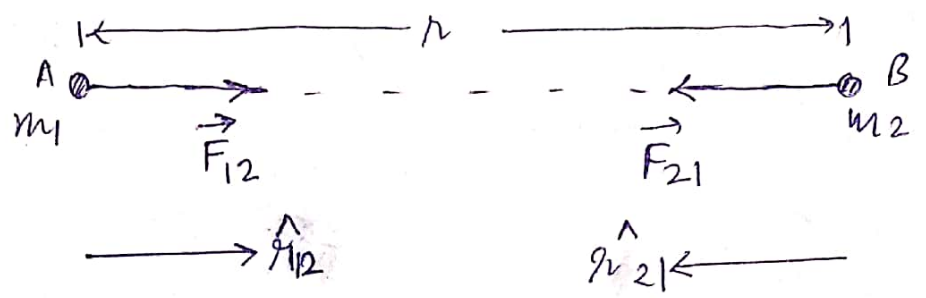
(ii) The tides are formed in oceans due to the gravitational force of attraction between the moon and sea water.

(iii) The times of solar and lunar eclipses calculated on the basis of the law of gravitation are found to be accurately.

(iv) The orbits and periods of revolution of artificial satellites can be found on the basis of the law of Gravitation..

(v) The value of g varies from place to place on the surface of the earth according to the law of gravitation.

Vector form of the Law of gravitation :–



Consider two particles A and B of masses m_1 & m_2 and seperated by distance r

Let $\hat{r}_{12}$ = Unit vector from A to B

$\hat{r}_{21}$ = Unit vector from B to A

$\vec{F}_{12}$ = Gravitational force exerted on A by B.

$\vec{F}_{21}$ = Gravitational force exerted on B by A

In vector form, Newton's law of Gravitation can be expressed as.

$$\vec{F_{12}} = -G\frac{m_1 m_2}{r^2}\hat{r}_{21} = -G\frac{m_1 m_2}{r^3}\cdot\vec{r_{21}}$$

The negative sign shows that the direction of force $\vec{F_{12}}$ is opposite to that of $\hat{r}_{21}$.

i.e. "The gravitational force is attractive in nature so that m_1 is attracted towards m_2."

Similarly. $$\vec{F_{21}} = -\frac{G m_1 m_2}{r^2}\hat{r}_{12} = -G\frac{m_1 m_2}{r^3}\vec{r_{12}}$$

Here $\hat{r}_{21} = \frac{\vec{r_{21}}}{|\vec{r_{21}}|} = \frac{\vec{r_{21}}}{r}$, $\hat{r}_{12} = \frac{\vec{r_{12}}}{|\vec{r_{12}}|} = \frac{\vec{r_{12}}}{r}$

But $\hat{r}_{21} = -\hat{r}_{12}$

Hence $\boxed{\vec{F_{21}} = -\vec{F_{12}}}$

Thus the vector form of the law of gravitation shows that the gravitational forces acting between two particles forms action and reaction pair.

As $\vec{F_{12}}$ and $\vec{F_{21}}$ are directed towards the centres of the two particles. so gravitational force is "a central force"

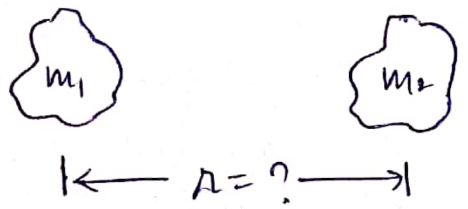
Properties of Gravitational Force :-

1. Gravitation force is always attractive in nature.
2. It is independent of the medium between the particles.
3. The mutual gravitational force between two bodies are equal and opposite i.e they form action and reaction pair. Hence gravitation forces obey Newton's third law of motion.
4. It is strictly holds for point masses i.e It is applicable for interplanetary to interatomic distances.
5. It is a central force. Its magnitude depends only on r and has no angular dependence. Thus the gravitational force has "spherical symmetry"
6. The gravitation force is a conservative force
7. It is the weakest force in the four fundamental forces

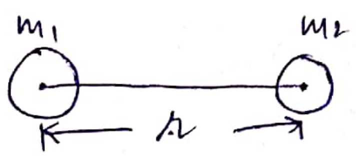
$$F_G < F_{W.N} < F_{E.M} < F_{S.N}.$$

8. Gravitation force is two-body interaction. i.e Gravitation force between two particles is independent of the presence of other bodies. So, the principle of superposition is valid.

NOTE :-



Not Applicable
(Non-Uniform mass)



(Applicable)
(Unique value of r)
(Uniform mass)

PRINCIPLE OF SUPERPOSITION OF GRAVITATIONAL FORCES :-

According to the principle of superposition, the gravitational force between two mass acts indepently and not affected by the presence of other bodies. "Hence the resultant gravitational force acting on a particle due to a number of masses is the vector sum of the gravitational forces exerted by the individual masses on the given particle."

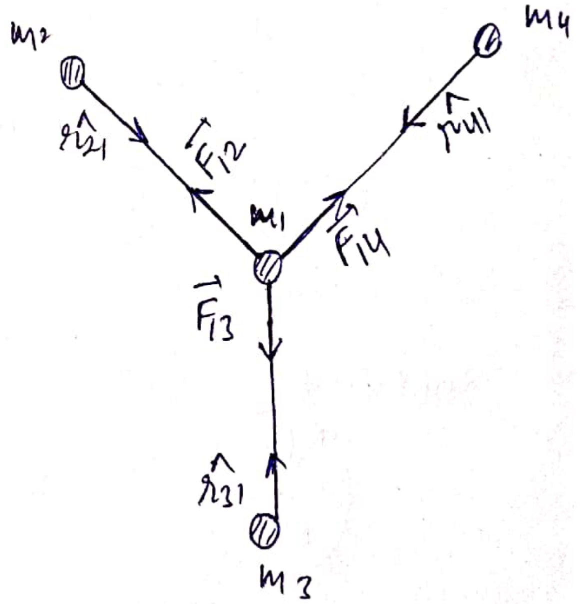
Mathematically

$$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots + \vec{F_n}$$

$$\boxed{\vec{F_R} = \sum_{i=1}^{n} \vec{F_i}}$$

where $\vec{F_1}, \vec{F_2}, \vec{F_3}, \ldots \vec{F_n}$ are the gravitational forces exerted by n individiual masses $m_1, m_2 \ldots m_n$ on the particle of mass m. Each force is determined by the law of gravitation.

Illustration :- The net force on mass m_1 due to masses m_2, m_3 and m_4.



$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}}$$

where $\vec{F_{12}} = -G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21}$

$$\vec{F_{13}} = -G \frac{m_1 m_3}{r_{31}^2} \hat{r}_{31}$$

and $\vec{F_{14}} = -G \frac{m_1 m_4}{r_{41}^2} \hat{r}_{41}$

$$\boxed{\vec{F_1} = -G \left[\frac{m_1 m_2}{r_{21}^2} \hat{r}_{21} + \frac{m_1 m_3}{r_{31}^2} \hat{r}_{31} + \frac{m_1 m_4}{r_{41}^2} \hat{r}_{41} \right]}$$

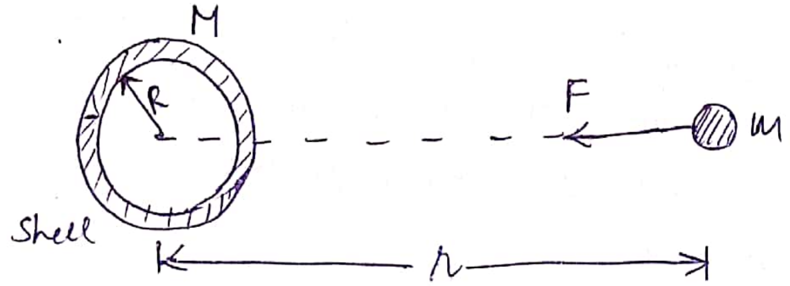
Newton's shell Theorem for the gravitational force.

This theorem gives gravitational force on a point mass due to a spherical shell or a solid sphere. It can be stated as follows.

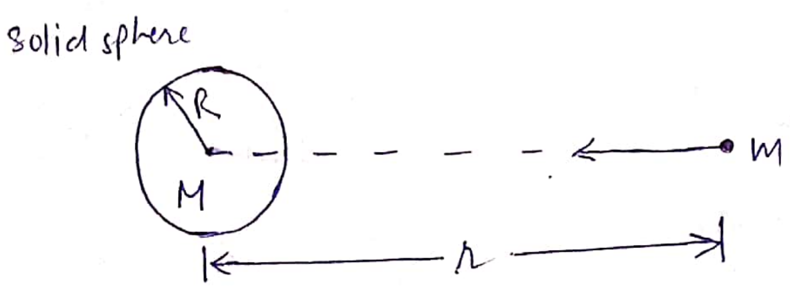
If point mass lies outside the spherical shell / sphere.

If a point mass lies outside a uniform spherical shell / sphere with a spherically symmetric internal mass distribution.

The shell / sphere attracts the point mass toward its centre if the entire mass of the sphere or shell were concentrated at its centre.



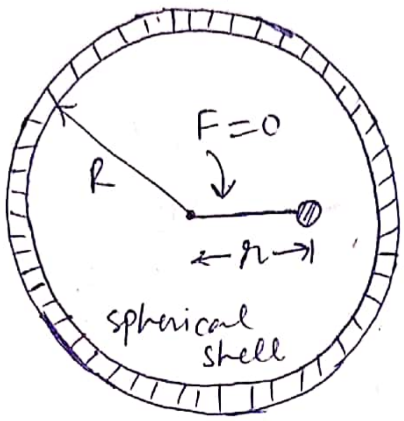
$$\boxed{F = \frac{GMm}{r^2}, \quad r \geq R}$$



$$\boxed{F = \frac{GMm}{r^2}, \quad r \geq R}$$

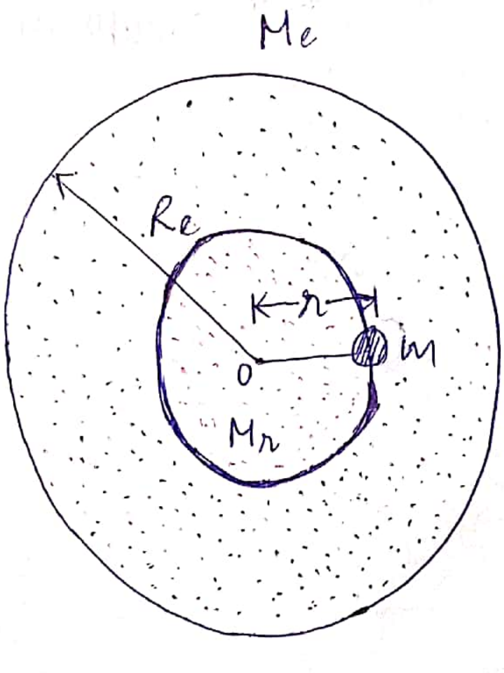
(ii) If a point mass lies inside a sphere / shell :

If a point mass lies inside a uniform spherical shell, the gravitational force on the point mass is zero. Because various regions of spherical shell attract the point mass inside it in various directions. These forces cancel each other completely.



$$\boxed{F = 0, \ r < R}$$

* If a point mass lies inside a homogenous solid sphere, the force on the point mass acts towards the centre of the sphere. This force is exerted by the spherical mass situated interior to the point mass.



$$F = \frac{GM_r \cdot m}{r^2}$$

$$\therefore M_r = \frac{4}{3}\pi r^3 \cdot \rho$$

$$M_r = \underbrace{\frac{4}{3}\pi R_e^3 \cdot \rho}\cdot \frac{r^3}{R_e^3}$$

$$M_r = M_e \cdot \frac{r^3}{R_e^3}$$

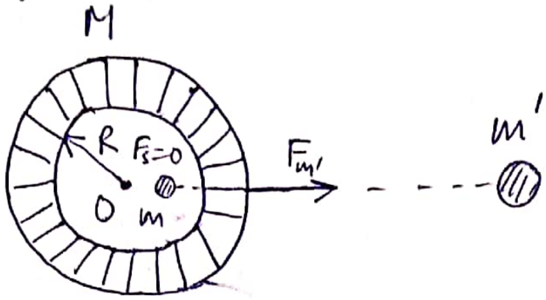
$$F = \frac{GM_e m \cdot r^{\cancel{3}}}{\cancel{r^2} \cdot R_e^3}$$

$$\boxed{F = \frac{GM_e m}{R_e^3} \cdot r}$$

$(r < R_e)$

Is Gravitational shielding possible?

Gravitational shielding is not possible, Although the gravitational force on a particle inside a shell is zero. yet the shell does not shield the other bodies outside it which exerts the gravitational force on particle lying inside it. Thus gravitation shielding is not possible



$F_{shell} = 0$ $\quad r < R$

$F_{m'} \neq 0$ $\quad$ Therefore gravitation shielding is not possible.

Q÷ Calculate the force of attraction between two balls each of mass 1 kg when their centre are 10 cm apart. Given $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$.

$m_1 = m_2 = 1\ kg$, $\quad r = 10\ cm = 10 \times 10^{-2}\ m$

$r = 0.10\ m.$

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = 6.67 \times 10^{-11} \times \frac{1 \times 1}{(0.10)^2}$$

$$F = \frac{6.67 \times 10^{-11}}{(10 \times 10^{-2})^2} = \frac{6.67 \times 10^{-11}}{100 \times 10^{-4}}$$

$$\boxed{F = 6.67 \times 10^{-9}\ N}$$

Scanned by CamScanner

Q- The mass of planet jupiter is 1.9×10^{27} kg and that of the sun is 1.99×10^{30} kg. The mean distance of the jupiter from the sun is 7.8×10^{11} m. Calculate the gravitational force which the sun exerts on jupiter. Assuming that jupiter moves in a circular orbit around the sun. Calculate the speed of jupiter.

$$m_J = 1.9 \times 10^{27} \text{ kg}$$

$$m_S = 1.99 \times 10^{30} \text{ kg}$$

$$r = 7.8 \times 10^{11} \text{ m}$$

$$F = G \frac{m_J m_S}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times 1.99 \times 10^{30}}{(7.8 \times 10^{11})^2}$$

$$F = \frac{25.219 \times 10^{46}}{60.84 \times 10^{22}}$$

$$F = \frac{25.219 \times 10^{46}}{6.084 \times 10^{23}}$$

$$\boxed{F = 4.145 \times 10^{23} \text{ N}}$$

If jupiter moves on a circular orbit.

$$F = \frac{m_J v^2}{r}$$

$$\frac{4.145 \times 10^{23} \times 7.8 \times 10^{11}}{1.9 \times 10^{27}} = v^2$$

$$\frac{32.331}{1.9} \times 10^{7} = v^2 \Rightarrow v^2 = 17.016 \times 10^{7}$$

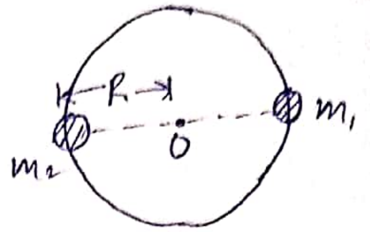
$$v = \sqrt{1.7016 \times 10^{8}}$$

$$\boxed{v_J = 1.304 \times 10^{4} \text{ m/s}}$$

Q- Two particles, each of mass m go round a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

$$m_1 = m_2 = m$$

$$r = R + R$$
$$r = 2R$$



(Binary system)

$$F_G = G\frac{m_1 m_2}{r^2}$$

$$F_G = G\frac{m \cdot m}{(2R)^2}$$

$$F_G = G\frac{m^2}{4R^2}$$

As both the particle moves along a circular path of radius R therefore the centripetal force on individual particle is given by

$$F_c = \frac{mv^2}{R}$$

For equillibrium $F_G = F_c$

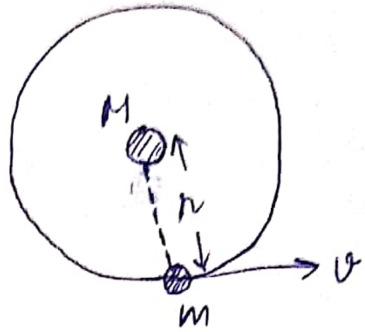
$$\frac{\cancel{m}v^2}{\cancel{R}} = \frac{Gm^{\cancel{2}}}{4R^{\cancel{2}}}$$

$$v^2 = \frac{GM}{4R}$$

$$\boxed{v = \sqrt{\frac{GM}{4R}}}$$

Q- If a body of mass m revolves around a planet of mass M along a circular path of radius r. If T is the time period of revolving body. Then prove that mass of planet is

$$M = \frac{4\pi^2 r^3}{GT^2}$$



Proof Force of Gravitation on the body is

$$F = \frac{Gm \cdot M}{r^2}$$

centripetal force required to revolved the body of mass m around the circular path.

$$F_c = \frac{mv^2}{r}$$

$$F_c = F \quad \text{(For equilibrium)}$$

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{G\cancel{m} \cdot M}{r^{\cancel{2}}}$$

$$v^2 = \frac{GM}{r} \quad \text{------ (i)}$$

If the time period of body is T then

$$v = \frac{\text{circumference of circular path (Distance)}}{\text{Time period.}}$$

$$v = \frac{2\pi r}{T} \quad \text{------ (ii)}$$

Put the value of eqn (ii) in eqn (i)

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\boxed{M = \frac{4\pi^2 r^3}{GT^2}}$$

Q- The mean orbital radius of earth around the sun is 1 AU. Calculate the mass of the sun. Given $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$$M_s = \frac{4\pi^2 r^3}{GT^2}$$

$r = 1\ AU$ (1 AU = mean distance between earth and sun)

$r = 1.5 \times 10^8\ km$

$1\ AU = 1.5 \times 10^8\ km$

$r = 1.5 \times 10^{11}\ m$

Time period of earth around the sun

$T = 365.2425$ days.

$T = 365.2425 \times 24 \times 60 \times 60$

$T = 3.156 \times 10^7$ sec.

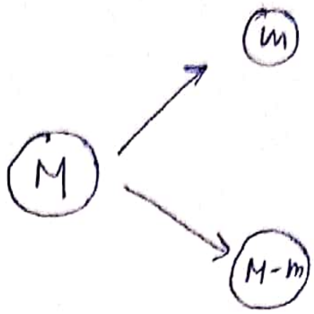
$$M_s = \frac{4 \times (3.141)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (3.156 \times 10^7)^2}$$

$$\boxed{M_s = 2.01 \times 10^{30}\ kg}$$

Q- A mass M is broken into two parts of masses m_1 and m_2. How are m_1 and m_2 related so that force of gravitational attraction between the two parts is maximum ?

Let $m_1 = m$ $\qquad m_1 + m_2 = M$

$m_2 = M - m$



$$F = G\frac{m_1 m_2}{r^2}$$

$$F = G\frac{m(M-m)}{r^2}$$

$$F = \frac{G}{r^2}(M.m - m^2). \quad \text{——} \quad (i)$$

Differentiate eq^n (i) w.r.t m

$$\frac{dF}{dm} = \frac{G}{r^2}\left[\frac{d}{dm}(M.m - m^2)\right]$$

$$\frac{dF}{dm} = \frac{G}{r^2}\left[\frac{d}{dm}M.m - \frac{d}{dm}m^2\right].$$

$$\frac{dF}{dm} = \frac{G}{r^2}[M.1 - 2m].$$

$$\frac{dF}{dm} = \frac{G}{r^2}[M - 2m].$$

For maximum gravitational force $\frac{dF}{dm} = 0$.

$$0 = \frac{G}{r^2}[M - 2m]$$

$$M - 2m = 0$$

$$M = 2m$$

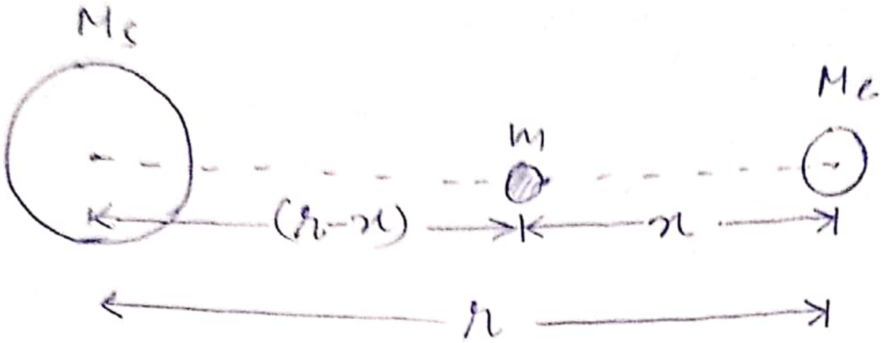
$$\boxed{m = \frac{M}{2}}$$

$$\boxed{m_1 = \frac{M}{2}}$$

$$m_2 = M - m = M - \frac{M}{2}$$

$$\boxed{m_2 = \frac{M}{2}}$$

Q- How far from earth must a body be along a line towards the sun so that the sun's gravitational pull on it balances that of the earth. Distance between sun and earth's centre is 1.5×10^{10} km. Mass of sun is 3.24×10^5 times mass of earth



Gravitational force due to sun on the body of mass m.

$$F_s = \frac{G M_s \cdot m}{(r-x)^2}$$

Gravitational force due to earth on the body

$$F_e = \frac{G M_e \cdot m}{x^2}$$

For the body to be in Gravitational balance

$$F_s = F_e$$

$$\frac{\cancel{G} M_s \cdot \cancel{m}}{(r-x)^2} = \frac{\cancel{G} M_e \cancel{m}}{x^2}$$

$$\frac{(r-x)^2}{x^2} = \frac{M_s}{M_e}$$

$$\left(\frac{r-x}{x}\right)^2 = \frac{M_s}{M_e}$$

$$\left(\frac{r}{x} - \frac{x}{x}\right)^2 = \frac{M_s}{M_e}$$

$$\left(\frac{r}{x} - 1\right)^2 = \frac{M_s}{M_e}$$

$$\frac{r}{x} - 1 = \sqrt{\frac{M_s}{M_e}}$$

$$\frac{r}{x} = 1 + \sqrt{\frac{M_s}{M_e}}$$

Given $r = 1.5 \times 10^{10}$ km

$r = 1.5 \times 10^{13}$ m

$M_s = 3.24 \times 10^5 \; M_e$

$$\frac{M_s}{M_e} = 3.24 \times 10^5$$

$$\frac{r}{x} = 1 + \sqrt{3.24 \times 10^5}$$

$$\frac{r}{x} = 1 + \sqrt{32.4 \times 10^4}$$

$$\frac{r}{x} = 1 + 5.7 \times 10^2$$

$$\frac{r}{x} = 1 + 570$$

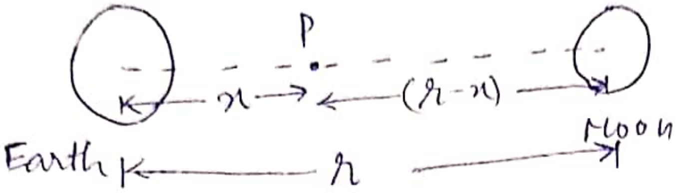
$$\frac{r}{x} = 571$$

$$x = \frac{r}{571} = \frac{1.5 \times 10^{10}}{571}$$

$$\boxed{x = 2.63 \times 10^7 \text{ km}}$$

Scanned by CamScanner

Q- The mass of Earth is 81 times that of the moon and the distance from the centre of Earth to that of the moon is about 4×10^5 km. Calculate the distance from the centre of the earth where the resultant gravitation force become zero. ?



let us consider at a distance x from the earth the gravitation force becomes zero.

The gravitational force at point P due to earth

$$F_E = \frac{G M_e \cdot m}{x^2} \quad \text{--- (1)}$$

m = mass of body which situated at point P

The gravitational force at point P due to moon.

$$F_M = \frac{G M_m \cdot m}{(r-x)^2}$$

For the resultant force at point P becomes zero.

$$F_E = F_M$$

$$\frac{G M_e \cdot \cancel{m}}{x^2} = \frac{G M_m \cdot \cancel{m}}{(r-x)^2}$$

$$\frac{(r-x)^2}{x^2} = \frac{M_m}{M_e}$$

Given $\quad M_e = 81 \; M_m$

$$\frac{M_m}{M_e} = \frac{1}{81}$$

$$\left(\frac{r-x}{x}\right)^2 = \frac{1}{81}$$

$$\frac{r-x}{x} = \sqrt{\frac{1}{81}}$$

$$\frac{r-x}{x} = \frac{1}{9}$$

$$x = 9r - 9x$$

$$10x = 9r$$

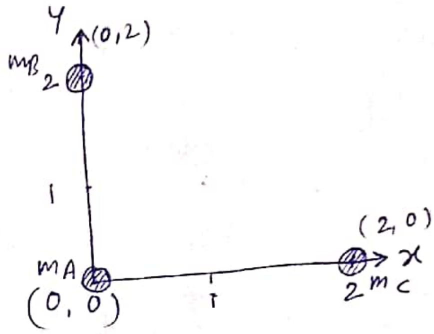
$$x = \frac{9}{10} r \qquad \text{Given } r = 4 \times 10^5 \text{ km}$$

$$x = \frac{9 \times 4 \times 10^{5}}{10}$$

$$x = 36 \times 10^4$$

$$\boxed{x = 3.6 \times 10^5 \text{ km}}$$

Q: Three identical point masses each of mass 1 kg lie in x-y plane at points (0, 0), (0, 2) and (2, 0). The net gravitational force on the mass which is at origin.



Given $m_A = m_B = m_C = 1$ kg.

Let particle A lies at origin, and particle B and C at y and x axis respectively.

Force on the mass m_A due to mass m_B

$$\vec{F}_{AB} = \frac{G m_A m_B}{r_{AB}^2} \hat{r}_{AB}$$

$$r_{AC} = x_2 - x_1 = 2 - 0 = 2$$

$$r_{AB} = y_2 - y_1 = 2 - 0 = 2$$

$$\vec{F}_{AB} = \frac{G m_A m_B}{r_{AB}^2} \cdot \hat{j}$$

$$\vec{F}_{AB} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(2)^2} \hat{j} = \frac{6.67 \times 10^{-11}}{4} \hat{j}$$

$$\boxed{\vec{F}_{AB} = 1.67 \times 10^{-11} \hat{j} \text{ N}}$$

Force on mass m_A due to mass m_C.

$$\vec{F}_{AC} = \frac{G m_A m_C}{r_{AC}^2} \cdot \hat{r}_{AC} = \frac{G m_A m_C}{r_{AC}^2} \hat{i} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(2)^2} \cdot \hat{i}$$

$$\boxed{\vec{F}_{AC} = 1.67 \times 10^{-11} \hat{i} \text{ N}}$$

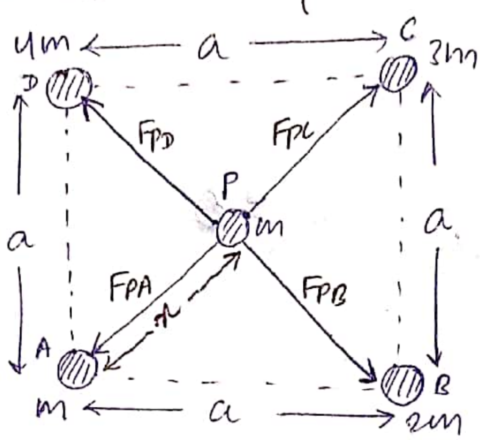
The net force on particle A

$$\vec{F} = \vec{F}_{AC} + \vec{F}_{AB}$$

$$\vec{F} = 1.67 \times 10^{-11}\, \hat{i} + 1.67 \times 10^{-11}\, \hat{j}$$

$$\boxed{\vec{F} = 1.67 \times 10^{-11} (\hat{i} + \hat{j})\ N}$$

Q- Four particles of mass m, 2m, 3m and 4m are kept in sequence at the corners of a square of side a. The magnitude of force of gravitation on a particle of mass m placed at the centre of the square?



Let the distance between the mass m situated at P and other masses which situated at the corners of the squares be x.

Force on the mass (mass) m at P due to mass m at point A

$$F_{PA} = \frac{Gm \cdot m}{x^2} = \frac{Gm^2}{x^2}$$

Force due mass 2m at P. is

$$F_{PB} = \frac{Gm(2m)}{x^2} = \frac{2Gm^2}{x^2}$$

Force due to mass 3m at P is

$$F_{PC} = \frac{Gm(3m)}{x^2} = \frac{3Gmm}{x^2} = \frac{3Gm^2}{x^2}$$

Force due to mass 4m at P is

$$F_{PD} = \frac{Gm(4m)}{x^2} = \frac{4Gm^2}{x^2}$$

As $F_{PC} > F_{PA}$, Net force along the line of mass 3m is

$$F_C = \frac{3Gm^2}{x^2} - \frac{Gm^2}{x^2}$$

$$F_C = \frac{2Gm^2}{x^2}$$

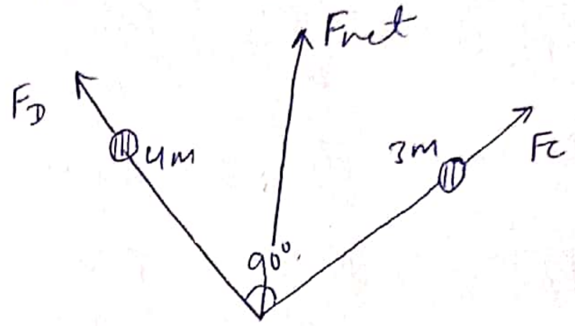
Scanned by CamScanner

and $F_{PD} > F_{PB}$, Net force along the line of mass 4m is

$$F_D = \frac{4Gm^2}{x^2} - \frac{2Gm^2}{x^2}$$

$$F_D = \frac{2Gm^2}{x^2}$$

Now Resultant of the forces F_C and F_D becomes



$$F_{net} = \sqrt{F_C^2 + F_D^2 + 2F_CF_D\cos\theta}$$

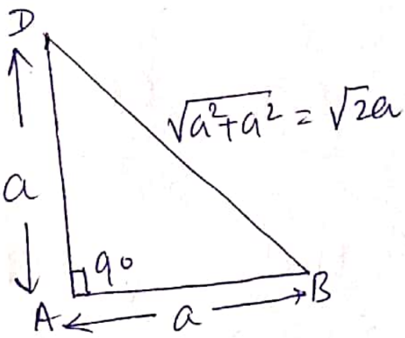
$$F_{net} = \sqrt{F_C^2 + F_D^2 + \underbrace{2F_CF_D\cos 90°}_{\to 0}}$$

$$F_{net} = \sqrt{F_C^2 + F_D^2}$$

$$F_{net} = \sqrt{\left(\frac{2Gm^2}{x^2}\right)^2 + \left(\frac{2Gm^2}{x^2}\right)^2}$$

$$\left[\text{let } \frac{Gm^2}{x^2} = K\right]$$

$$F_{net} = \sqrt{(2K)^2 + (2K)^2}$$



$$F_{net} = \sqrt{4K^2 + 4K^2} = \sqrt{8K^2}$$

$$F_{net} = 2\sqrt{2}K$$

$$BD = 2x$$

$$2x = \sqrt{2}a$$

$$x = \frac{\sqrt{2}a}{2}$$

$$x = \frac{a}{\sqrt{2}}$$

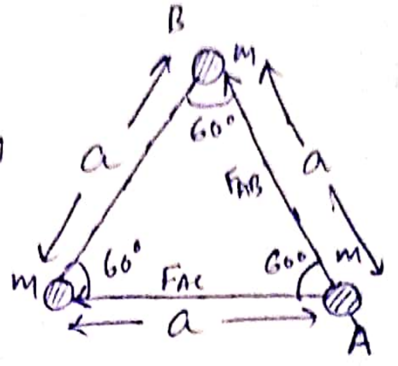
$$F_{net} = \frac{2\sqrt{2}Gm^2}{x^2}$$

$$F_{net} = \frac{2\sqrt{2}Gm^2}{\left(\frac{a}{\sqrt{2}}\right)^2}$$

$$F_{net} = \frac{2\sqrt{2}Gm^2}{a^2/2}$$

$$\boxed{F_{net} = \frac{4\sqrt{2}Gm^2}{a^2}}$$

Q- Three identical masses each are situated at the verticies of a triangle (equillateral) of side a. Find the force experienced by one of the mass due to the other two masses?



Gravitational force on mass m situated at A due to mass m situated at B.

$$F_{AB} = G\frac{m \cdot m}{a^2} = \frac{Gm^2}{a^2}$$

Similarly due to mass m situated at C.

$$F_{AC} = G\frac{m \cdot m}{a^2} = \frac{Gm^2}{a^2}$$

Net force on particle at A due to F_{AB} and F_{AC} is

$$F_{net} = \sqrt{F_{AB}^2 + F_{AC}^2 + 2F_{AB}F_{AC}\cos\theta}$$

let $\frac{Gm^2}{a^2} = K$

$$F_{net} = \sqrt{K^2 + K^2 + 2K \cdot K \cos 60^\circ}$$

$\frac{1}{2}$

$$F_{net} = \sqrt{K^2 + K^2 + \cancel{2}K^2 \cdot \frac{1}{\cancel{2}}}$$

$$F_{net} = \sqrt{3K^2}$$

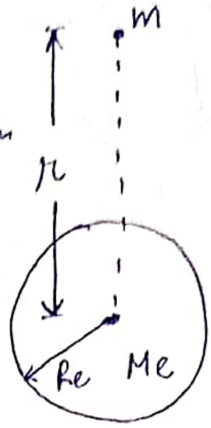
$$F_{net} = \sqrt{3}K$$

$$\boxed{F_{net} = \frac{\sqrt{3}Gm^2}{a^2}}$$

The gravitational force exerted by the earth on a point mass m located above, below and on the surface of the earth.

(i) At a point above the earth surface.

Force of Gravity on the mass m exerted by earth. According to shell theorem



$$\boxed{F = G\frac{M_e \cdot m}{r^2}} \qquad (r > R_e)$$

where.
M_e = mass of earth.

we consider earth as a solid sphere of radius R_e

(V_e) Volume of the Earth $= \frac{4}{3}\pi R_e^3$

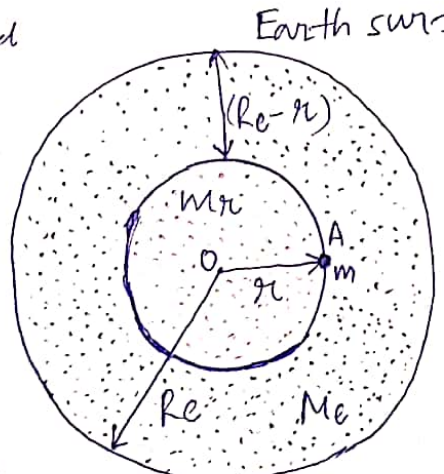
$$\text{Density of Earth}(\rho) = \frac{\text{Mass of earth } (M_e)}{\text{Volume of earth } (V_e)}$$

$$\rho = \frac{M_e}{\frac{4}{3}\pi R_e^3}$$

$$\boxed{M_e = \frac{4}{3}\pi R_e^3 \cdot \rho}$$

(ii) At a point below the earth's surface :-

Suppose a point mass m is situated at a point A at a depth $(R_e - r)$ below the earth's surface. Thus the point A lies outside the sphere of radius r and inside the shell of thickness $(R_e - r)$



According to shell theorem. The outer shell exerts no force on the mass m kept at point A... Therefore inner sphere of radius r exerts a force on the point mass m at point A as if its mass M_r is concentrated at the centre.

Me → Mass of Earth.
Re → Radius of Earth
m → mass of point mass situated at point A
mr → mass of inner sphere
r → radius of inner sphere.

Gravitation force exerted by the inner sphere on the point mass m.

$$F = G\frac{m_r \cdot m}{r^2} \quad \text{——— (1)}$$

But the mass of inner sphere is given by

$$\underbrace{m_r}_{\text{mass of inner sphere}} = \underbrace{\frac{4}{3}\pi r^3}_{\text{Vol}^m \text{ of inner sphere}} \cdot \underbrace{\rho}_{\text{Density of earth}} \quad \text{——— (2)}$$

Multiply by Re^3 in the numerator and Denominator of eqn (2)

$$m_r = \frac{4}{3}\pi r^3 \cdot \rho \cdot \frac{Re^3}{Re^3} \quad \xrightarrow{\text{By. Rearranging}}$$

$$m_r = \underbrace{\frac{4}{3}\pi Re^3 \cdot \rho}_{\text{Mass of Earth. Me}} \left(\frac{r^3}{Re^3}\right)$$

$$m_r = Me \cdot \frac{r^3}{Re^3}$$

Put the value of m_r in eqn (1) we get.

$$F = G \cdot \frac{\left(Me \cdot \frac{r^3}{Re^3}\right) \cdot m}{r^2}$$

$$F = \frac{G \cdot M_e \cdot \cancel{r^2} \cdot m}{R_e^3 \cdot \cancel{r^2}}$$

$$\boxed{F = \frac{G M_e m}{R_e^3} \cdot r}$$ (For $r < R_e$) ..... (3).

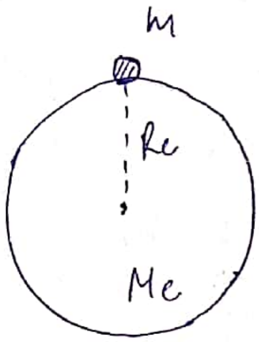
OR.

$$F = \frac{G M_e m}{R_e^2} \left(\frac{r}{R_e} \right)$$ (Force on the surface of the earth)

$$\boxed{F = F_e \cdot \left(\frac{r}{R_e} \right)}$$ $\left[\because F_e = \frac{G M_e m}{R_e^2} \right]$

(iii) At a point on the surface of the earth :-

If the point mass m is situated on the earth's surface then ($r = R_e$)



The gravitation force on mass m is given by

$$\boxed{F = \frac{G M_e \cdot m}{R_e^2}}$$ ($r = R_e$)

OR.

$$F = \frac{G M_e m}{R_e^3} \cdot r$$ if $r = R_e$

$$F = \frac{G M_e m}{R_e^{\cancel{3}\,2}} \cdot \cancel{R_e}$$

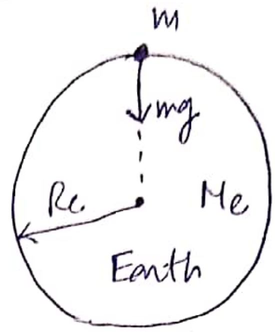
$$\boxed{F = \frac{G M_e m}{R_e^2}}$$

Relation between acceleration due to gravity g in terms of mass of earth Me and gravitation constant G.

OR

Relation between g and G ?

Consider the earth to be a sphere of mass Me and radius Re. Suppose a body of mass m is lying on earth's surface.



The gravitation force exerted by the earth on the mass m.

$$F = \frac{G M_e \cdot m}{R_e^2} \quad \text{——— (1)}$$

This force of gravity (F) produce an acceleration g in the body of mass m known as acceleration due to gravity.

From Newton's second law.

$$F = mg \quad \text{——— (2)}$$

From eqn (i) and (2)

$$\cancel{m} g = \frac{G M_e \cdot \cancel{m}}{R_e^2}$$

$$\boxed{g = \frac{G M_e}{R_e^2}} \quad \text{——— (3)}$$

G, Me and Re all are constants. Therefore the value of g is independent of the mass, size and shape of body falling under gravity.

If ρ is the density of Earth. Then the mass M_e is given as

$$M_e = \frac{4}{3}\pi R_e^3 \rho.$$

Put it into the eqn (2) we get

$$g = \frac{G\cdot\left(\frac{4}{3}\cdot\pi R_e^3 \rho\right)}{R_e^2}$$

$$\boxed{g = \frac{4}{3}\pi \rho G R_e.}$$

Mass and density of the Earth :

Mass of the Earth : As the accleration due to gravity

$$g = \frac{GM_e}{R_e^2}$$

[NOTE - The weight of earth first determined by cavendish. because he is the first person who determines the value of G.]

The mass of Earth be

$$M_e = \frac{gR_e^2}{G}$$

knowing the values of g, R_e and G we can determined the mass of earth

As $g = 9.8\ m/s^2$, $R = 6370\ km = 6370 \times 10^3\ m = 6.37 \times 10^6\ m$

$$M_e = \frac{gR_e^2}{G}$$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$$M_e = \frac{9.8 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$M_e = \frac{9.8 \times 40.58 \times 10^{12}}{6.67 \times 10^{-11}} = \frac{397.684}{6.67} \times 10^{23}$$

$$M_e = 5.97 \times 10^{24}\ kg$$

$$\boxed{M_e \cong 6 \times 10^{24}\ kg}$$

Scanned by CamScanner

Average density of earth :- If the earth is taken as a sphere of density ρ then its mass would be.

$$M_e = \frac{4}{3}\pi R_e^3 \cdot \rho \quad \text{---- (1)}$$

From the acceleration due to gravity

$$g = \frac{GM_e}{R_e^2}$$

$$M_e = \frac{gR_e^2}{G}$$

Put it into eqn (1)

$$\frac{gR_e^{\cancel{2}}}{G} = \frac{4}{3}\pi R_e^{\cancel{3}} \cdot \rho$$

$$\boxed{\rho_{avg} = \frac{3g}{4\pi G R_e}}$$

if $g = 9.8\ m/s^2$, $G = 6.67\times10^{-11}\ Nm^2/kg^2$,
$R_e = 6.37\times10^6\ m$ (6370 km)

$$\rho = \frac{3\times9.8}{4\times3.142\times6.67\times10^{-11}\times6.37\times10^6}$$

$$\rho = \frac{29.4}{533.99\times10^{-5}}$$

$$\rho = 0.05497\times10^5$$

$$\rho = 5497\ kg/m^3$$

$$\boxed{\rho_{avg} \simeq 5500\ kg/m^3}$$

[The density of upper layer of earth is 2700 kg/m³ while the density of inner layer is much larger than average value of 5500 kg/m³. But the average value becomes 5500 kg/m³.]

Q: You are given the following data: $g = 9.8\ m/s^2$ $R_e = 6.37 \times 10^6\ m$. The distance to the moon $r = 3.84 \times 10^8\ m$ and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth in two different ways?

(i)
$$g = 9.8\ m/s^2,\quad R_e = 6.37 \times 10^6\ m$$
$$r = 3.84 \times 10^8\ m \quad T = 27.3\ \text{days}$$

$$g = \frac{GM_e}{R_e^2}$$

$$M_e = \frac{gR_e^2}{GM_e}$$

[Refer to article "Mass of the Earth"]

$$M_e = 5.97 \times 10^{24}\ kg$$

$$\boxed{M_e \simeq 6 \times 10^{24}\ kg}$$

(ii) From kepler's law of period. (Mass of satelite)

$$M_e = \frac{4\pi^2 r^3}{GT^2}$$

$$M_e = \frac{4 \times (3.14)^2 \times (3.84 \times 10^8)^3}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

$$M_e = \frac{4 \times 9.86 \times 14.75 \times 10^{24} \times 3.84}{6.67 \times 10^{-11} \times (2.36 \times 10^6)^2}$$

$$M_e = \frac{581.74 \times 10^{24} \times 3.84}{6.67 \times 5.57 \times 10^{-11} \times 10^{12}}$$

$$M_e = \frac{2.234 \times 10^{27}}{3.72 \times 10^2}$$

$$M_e = 6.02 \times 10^{24}\ kg$$

$$\boxed{M_e \simeq 6 \times 10^{24}\ kg}$$

Q÷ If the earth were made of lead of relative density 11.3. what then would be the value of acceleration due to gravity on the surface of earth? Radius of the earth = 6.4×10^6 m and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

$$\text{Relative density} = \frac{\text{density of lead (Earth)}}{\text{density of water}}$$

$$\rho_R = 11.3$$
$$\rho_W = 10^3 \ kg/m^3$$

density of Earth $\rho = \rho_R \times \rho_W$

$$\rho = 11.3 \times 10^3 \ kg/m^3$$

$$g = \frac{GMe}{Re^2} \qquad [\because Me = \tfrac{4}{3}\pi Re^3 \cdot \rho.]$$

$$g = \frac{G \cdot 4\pi Re^{\cancel{3}} \cdot \rho}{3 \cancel{Re^2}}$$

$$g = \frac{4}{3}\pi G Re \cdot \rho.$$

$$g = \frac{4}{3} \times \frac{22}{7} \times 6.67 \times 10^{-11} \times 6.4 \times 10^6 \times 11.3 \times 10^3.$$

$$\boxed{g = 22.21 \ m/s^2}$$

Q- The acceleration due to gravity at the moon's surface is $1.67 \ m/s^2$. If the radius of the moon is 1.74×10^6 m. Calculate the mass of moon.

$$g = \frac{GM_m}{R_m^2}$$

$$M_m = \frac{gR_m^2}{G}$$

$g = 1.67\ m/s^2$, $R_m = 1.74 \times 10^6\ m$

$$M_m = \frac{1.67 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$\boxed{M_m = 7.58 \times 10^{22}\ kg}$$

Q- A body weighs 90 kgf (killogram-force) on the surface of the earth. How much will it weigh on the surface of Mars whose mass is 1/9 and the radius is 1/2 of that of the earth.

The acceleration due to gravity on the surface of the earth

$$g_e = \frac{GM_e}{R_e^2} \quad \text{--- (i)}$$

The acceleration due to gravity on the surface of mars is

$$g_M = \frac{GM_m}{R_m^2} \quad \text{--- (ii)}$$

Dividing eq^n (ii) by (i) we get.

$$\frac{g_m}{g_e} = \frac{\frac{GM_m}{R_m^2}}{\frac{GM_e}{R_e^2}} = \frac{GM_m \cdot R_e^2}{GM_e R_m^2}$$

$$\frac{g_m}{g_e} = \left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)^2$$

$$M_m = \frac{1}{9} M_e \Rightarrow \frac{M_m}{M_e} = \frac{1}{9}$$

$$R_m = \frac{1}{2} R_e \Rightarrow \frac{R_e}{R_m} = 2$$

$$\frac{g_m}{g_e} = \frac{1}{9} \times (2)^2 \qquad \boxed{g_m = \frac{4}{9} g_e}$$

Weight of the body on the mars is:

$$W_M = mg_m$$

$$W_m = \frac{4}{9} mg_e$$

(weight on Earth)

$$W_M = \frac{4}{9} W_e$$

given $W_e = 90$ kg.f

$$W_m = \frac{4}{\cancel{9}} \times \cancel{90}^{10}$$

$$\boxed{W_m = 40 \text{ kg.f}}$$

Q- If the radius of the earth shrinks by 2%, mass remaining constant, then how would the value of acceleration due to gravity change?

Acceleration due to gravity on the surface of the earth is given by

$$g = \frac{GMe}{Re^2}$$

Taking log both sides we get

$$\log g = \log(GMe) - \log(Re)^2$$

$$\log g = \log G + \log Me - 2 \log Re.$$

differentiate the above eqn. we get

$$\frac{dg}{g} = 0 + 0 - 2\frac{dRe}{Re} \quad [G, Me \text{ are constant}]$$

$$\frac{dg}{g} = -2\frac{dRe}{Re}$$

Given $\frac{dRe}{Re} = 2\% = \frac{-2}{100}$ (decrease by 2%)

$$\frac{dg}{g} = -2\left(\frac{-2}{100}\right)$$

$$\frac{dg}{g} = +\frac{4}{100}$$

% increase in value of g.

$$\frac{dg}{g} \times 100 = \frac{4}{\cancel{100}} \times \cancel{100}$$

$$\boxed{\% \frac{dg}{g} = 4\%} \text{ (Increase)}$$

Q- Acceleration due to gravity on the moon is 1/6 of acceleration due to gravity on earth. If the ratio of densities of earth ρ_e and moon ρ_m is $\rho_e / \rho_m = 5/3$ then radius of moon R_m in terms of R_e will be ?

Acceleration due to gravity is given by

$$g_e = \frac{GM_e}{R_e^2} \qquad M_e = \frac{4}{3}\pi R_e^3 \cdot \rho_e.$$

$$g_e = \frac{G \cdot 4\pi R_e^{\cancel{3}} \rho_e}{3 \cancel{R_e^2}} = \frac{4}{3}\pi G R_e \rho_e.$$

similarly for moon

$$g_m = \frac{4}{3}\pi G R_m \cdot \rho_m$$

$$\frac{g_m}{g_e} = \frac{\cancel{\frac{4}{3}\pi G} R_m \rho_m}{\cancel{\frac{4}{3}\pi G} R_e \rho_e} = \frac{R_m \cdot \rho_m}{R_e \cdot \rho_e.}$$

$$g_m = \frac{1}{6} g_e, \quad \rho_e / \rho_m = 5/3$$

$$\frac{g_m}{g_e} = \frac{1}{6} \qquad \rho_m / \rho_e = 3/5$$

$$\frac{R_m}{R_e} = \left(\frac{g_m}{g_e}\right)\cdot\left(\frac{\rho_e}{\rho_m}\right)$$

$$\frac{R_m}{R_e} = \frac{1}{6} \times \frac{5}{3}$$

$$\frac{R_m}{R_e} = \frac{5}{18}$$

$$\boxed{R_m = 5/18\, R_e}$$